Displacement Demand Estimation of an Elevated Rectangular Reinforced Concrete Water Tank Using Nonlinear Time History Analysis: A Case Study

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Abstract—**This paper presents a research study aimed at understanding the displacement demand and the dynamic behavior of an elevated reinforced concrete rectangular water tank subjected to moderate and strong seismic events. The water tank was designed to resist gravity and wind loads and was subjected to ground acceleration records obtained from actual seismic events. The tank was analyzed in both empty and fully filled states using nonlinear time history analysis by providing eleven ground acceleration records as the dynamic lateral force. The results were then examined in terms of displacement demand, base shear, nonlinear behavior, and resonance of the tank. The study revealed that the displacement demand of a structure subjected to a seismic event is not solely determined by a single factor, but rather it is influenced by a combination of factors, including peak ground acceleration (PGA), duration, and the earthquake magnitude. Furthermore, the absence of plastic hinge formation under seismic events with a PGA around 0.1g indicates that, despite this type of water tanks in Sri Lanka not being explicitly designed to withstand seismic forces, they exhibit a robust resistance to earthquakes of this magnitude. As a result, it can be concluded that such tanks are unlikely to fail during seismic events of this magnitude.**

Keywords—Time history analysis, displacement demand, dynamic behavior, plastic hinges, ground acceleration

I. INTRODUCTION

Elevated water tanks serve a vital function within water storage and distribution systems, catering to the specific needs of various regions and communities. These tanks are available in a diverse range of shapes, sizes, and materials, designed to suit their intended purpose and regional requirements. Rectangular reinforced concrete tanks are a popular choice in many regions worldwide due to their space efficiency, low construction cost, ease of construction and maintenance. Various staging types are employed to support elevated RC water tanks, with frame staging being commonly used. Kumbhar et al. [1] stated that these elevated water tanks supported by the reinforced concrete staging are classified as inverted pendulum structures as most of the mass is concentrated on the top of the tank staging.

Seismic events generate ground accelerations and any structure that experiences those will gradually deform until it achieves its maximum displacement. This maximum deformation is known as the displacement demand of the structure, and it is a critical factor in determining the structural stability and safety of structures subjected to dynamic loadings. There are four primary approaches to analyze structures in practice and time history analysis which is a type of dynamic analysis is one of the best suited for studying the behavior and the displacement demand of structures subjected to dynamic loadings, considering material yielding, large deformations, and nonlinearities. It provides a step-by-step evaluation of a structure's dynamic behavior, using modal or direct integration methods to solve dynamic equilibrium equations, generally referred to as equations of motion. Soroushnia et al. [2] have studied and analyzed a sample of a reinforced concrete elevated water tank, with 900 cubic meters under one earthquake record using dynamic time history analysis. For three cases of completely filled, half filled, and empty tanks time history analysis was performed and failure modes and damages to structures were evaluated.

Currently, the US National Earthquake Information Center tracks roughly 20,000 earthquakes annually, or about 55 per day, around the world. Long-term records (dating back to around 1900) predict that there will typically be 16 significant earthquakes per year. Therefore, it is imperative to ensure that critical structures, such as water tanks, are functional following seismic events. Given that Sri Lanka is not located in an area prone to earthquakes, the infrastructure is typically not designed to withstand seismic forces. Seneviratne et al. [3] had analyzed over 300 records of earthquakes around Sri Lanka and found that the Mannar rift zone and Comorin ridge is the critical zone of influence capable of generating a seismic event of magnitude 6.9 at a 475-year return period at a depth of 10-15 km below the sea level. As such, seismicity in Sri Lanka can no longer be ignored. Uduweriya et al. [4] found that the Peak Ground Acceleration (PGA) is highest in the west coastal zone within a range of 0.05 to 0.1 g, with an approximate PGA value of 0.1 g in Colombo, using Probabilistic Seismic Hazard Assessment (PSHA) that considers seismic activities in the surrounding region.

Given the paramount significance of seismic analysis, it is imperative to investigate the behavior of structures exposed to seismic forces and to incorporate seismic design considerations into the structure if any weaknesses are uncovered. In order to improve the construction and design of these structures, it is essential to have a comprehensive understanding of the behavior of displacement demand. This paper will focus on the changes in displacement demand and the dynamic behavior of an elevated reinforced concrete water tank with respect to different ground acceleration records in the context of moderate and strong seismic events, utilizing nonlinear time history analysis.

II. Methodology

In this study, an elevated reinforced concrete water tank has been designed to resist the effects of gravity and wind loads. The designed structure is then subjected to nonlinear time history analysis to estimate the displacement demand that may result from ground accelerations caused by seismic events. The tank is analyzed under both empty and fully filled conditions, and the results are evaluated in terms of displacement demand, base shear, nonlinear behavior, and resonance of the tank. Etabs Ultimate [5] is primarily employed for structural design as it offers specialized tools for designing shear wall reinforcement, an option not available in SAP2000 [6]. Subsequently, the designed structure is modelled in SAP2000 for conducting time history analysis. This choice is influenced by SAP2000's user-friendly interface, which is advantageous for performing complex time history analyses.

A. Design of the Elevated Rectangular Water Tank

The elevated rectangular tank has been designed in accordance with its Ultimate Limit State (ULS), and Serviceability Limit State (SLS) in compliance with Eurocodes [7, 8]. However, seismic forces were not considered during the design process. The design requirements, material properties and the designed section sizes are outlined in Tab. 1 to 3.

TABLE I. Design Requirements

Design Requirements	Value
Capacity	300 m^3
Width of the Tank	7 m
Length of the Tank	10 m
Height of the Tank	5 m
Staging Height	12 m

TABLE II. Material Properties and Design Parameters

Material Properties & Design Parameters	Value
Concrete Class	30/37
Concrete - Density	25 kN/m^3
Concrete - Modulus of Elasticity	33 GPa
Steel Reinforcement - Characteristic Yield Strength	500 N/mm^2
Steel Reinforcement - Density	78.5kN/m^3
Steel - Modulus of Elasticity	200 GPa
Water Tightness Class	1
Basic Wind Speed	22 ms^{-1}
Exposure Condition	XC3/4

TABLE III. Member Sizes and Other Design Information

Description	Dimensions (mm)
Column (C1)	350 x 350
Main Beam (B1)	400 x 350
Bracing Beam (BB1)	400 x 350
Top Slab (TS)	150
Bottom Slab (BS)	300
Shear Wall (SW1)	350
Clear Cover	40

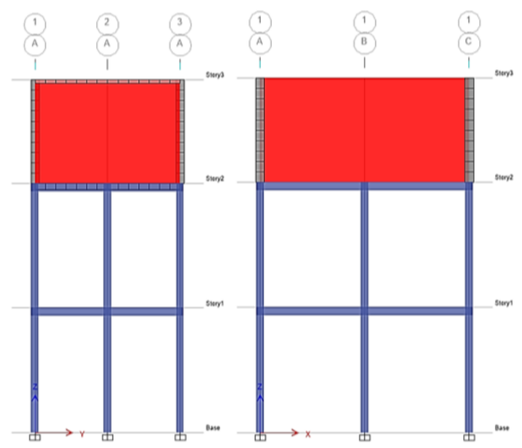

Fig. 1. Elevation views of the designed water tank

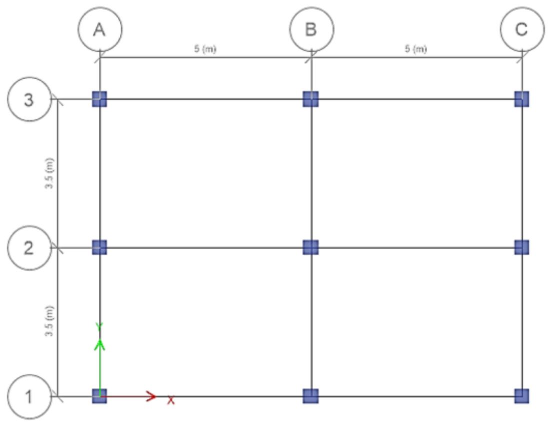

Fig. 2. Column layout of the water tank

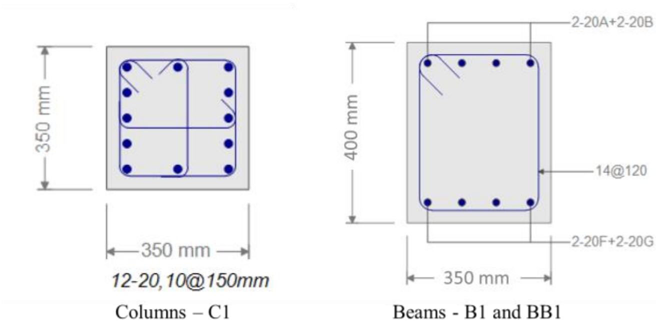

Fig. 3. Reinforcement details of the staging

B. Finite Element Model

The geometry of the tank and reinforcement details are shown in Fig. 1, 2 and 3. The finite element models of the structures were developed utilizing the SAP2000 program, as shown in Fig. 4. These models accurately represent the physical properties and geometry of the structure, including its materials, connections, and boundary conditions. The tank was modelled with the assumption that it is fixed at the base. For the purpose of the study, both the tank empty and full conditions were considered. A total of 22 combinations were studied for the tank under eleven different earthquake ground motions.

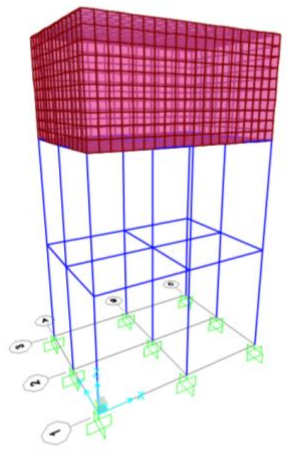

Fig. 4. Finite element model of the water tank

When the tank is fully filled, the mass is evenly distributed along the perimeter of the tank at the mid-level (14.5 m from the ground and 2.5 m from the tank base). The staging has been modelled with plastic hinges to accommodate for the non-linearity of the structure under both empty and full conditions, as depicted in Fig. 5. The default Moment (M3) hinges and axial load and moment (P-M2-M3) hinges available in SAP2000 were assigned for beams and columns respectively. The moment-rotation relationships and acceptance criteria for hinge performance levels were sourced from ATC 40 (Applied Technology Council) [9] and FEMA 356 guidelines (Federal Emergency Management Agency) [10]. Hinges were allocated at 0.05 and 0.95 of the relative distance along each frame element [11].

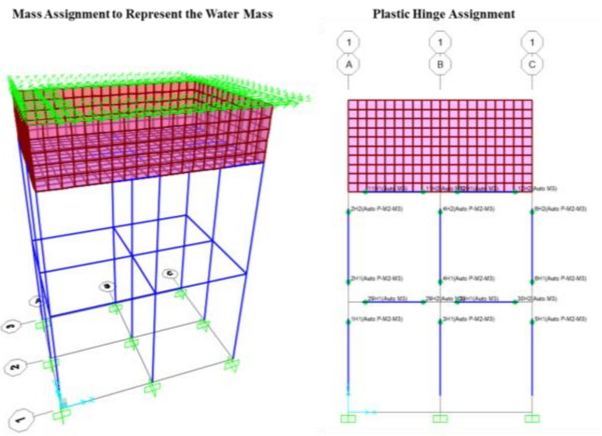

Fig. 5. Water mass and plastic hinge assignment

The natural frequencies of the first two modes of both empty and fully filled water tank were calculated using (1). The stiffness (k) was derived from the gradient of the pushover curves obtained through pushover analyses of the finite element model. The mass (m) was determined by considering the concrete and water masses. Tab. 4 presents the resulting values. The validity of the finite element model was confirmed by comparing the first two modes natural frequencies computed using (1) with those obtained by modal analysis of the finite element model as shown in section III. D.

$$\omega_n = (1/2\pi)\,(\sqrt{k/m}) \qquad (1)$$

TABLE IV. STIFFNESS, MASS AND NATURAL FREQUENCIES

Description	Stiffness (N/m)	Mass (kg)	Natural Frequency (Hz)
Empty Tank – Mode 1	5888828.5	298912.9	0.71
Empty Tank – Mode 2	6218281.0	298912.9	0.73
Fully Filled Tank – Mode 1	5618511.0	604417.4	0.49
Fully Filled Tank – Mode 2	5931836.4	604417.4	0.50

C. Selection of Ground Motions

Seismic ground motion records, also known as accelerograms or seismograms, are digital illustrations that depict the ground motion ensuing from an earthquake. These records provide precise information about the time-history of ground accelerations, velocities, and displacements during an earthquake. These accelerations serve as the input data for time-history analysis. The ground motions were sourced from the PEER Database [12] and were selected based on the following criteria. Six records of 0.1g PGA were chosen to conform to the Sri Lankan conditions, and five records with a PGA greater than 0.1 were selected from moderate and strong seismic events.

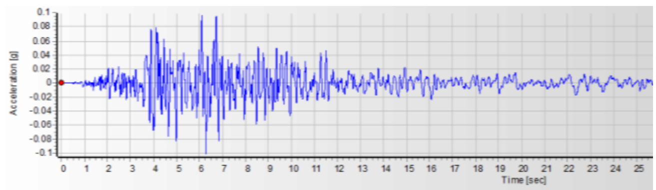

Fig. 6. Sample ground acceleration record (*M3-Westmoreland Earthquake*)

The ground acceleration records from moderate earthquakes with a magnitude between 5.0 and 5.9, and strong earthquakes with a magnitude between 6.0 and 6.9 were selected and the selected earthquakes are listed in Tab. 5 in ascending order based on their PGA. A sample acceleration record is shown in Fig. 6. The magnitude of each earthquake is expressed in the moment magnitude scale.

TABLE V. SELECTED GROUND MOTION RECORDS

EQ ID	Name	Magnitude	Duration (s)	PGA (g)
M1	Mammoth Lakes-04	5.70	40.00	0.098
M2	Anza (Horse Canyon)-01	5.19	10.31	0.099
M3	Westmorland	5.90	40.00	0.101
M4	Coyote Lake	5.74	26.83	0.102
M5	Coalinga-04	5.18	19.01	0.105
M6	Whittier Narrows-02	6.19	21.99	0.113

S1	Parkfield	6.53	26.20	0.248
S2	Imperial Valley-06	6.61	37.84	0.277
S3	San Fernando	6.06	36.68	0.382
S4	Mammoth Lakes-01	5.77	29.99	0.430
M7	Coalinga-05	6.19	21.73	0.575

D. Time History Analysis

Nonlinear time history analysis is a numerical approach employed to simulate the dynamic response of a structure under seismic or other dynamic loads. Unlike linear analysis, which fails to account for the nonlinear behavior of materials and components, this method provides a more accurate portrayal of a structure's deformation and response to realistic, nonlinear conditions such as material yielding, large displacements, and plastic hinge formation. In this research, the direct integration method is chosen as the solution type when conducting the time history analysis. The Hilber-Hughes-Taylor (HHT) method has been selected as the numerical integration technique for conducting time history analysis as it is effective when dealing with significant variations of stiffness and mass matrices during the analysis, such as in the presence of nonlinearities. Also, this method is proven to yield more stable base shear results compared to Newmark, and Chung & Hulbert Method [13]. The analysis was conducted incorporating P-delta nonlinearity, and assuming a damping of 5%.

III. RESULTS AND DISSCUSSION

The nonlinear time history analysis is performed on the both the empty and fully filled tanks. The results were documented and discussed in terms of displacement demand, base shear, frequency content and plastic hinge formation.

A. Displacement Demand

The displacement demand of the structure is the maximum expected displacement that the structure will experience during a given seismic event. Displacement demand comparative to the structure's capacity determines whether it can safely withstand the expected seismic forces and deformations. If the displacement demand exceeds the capacity, design modifications may be necessary to improve the structure's seismic performance, such as adding additional bracing or dampers, strengthening components, or altering the structural system.

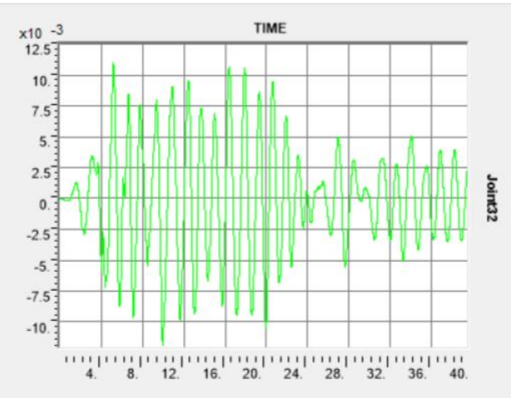

Fig. 7. Sample Displacement Plot Function. (*M3-Westmoreland Earthquake*)

The maximum displacement recorded by considering a node (node no. 32) located in the roof of the tank, under empty and fully filled conditions for each earthquake is shown in Tab. 6. The maximum value is taken from the displacement plot function, as shown in Fig. 7. When evaluating the empty state, the maximum displacement demand is attained for the earthquake S4, where the PGA is 0.43g and the duration is 29.99sec. Conversely, the minimum displacement demand is attained for the earthquake M2, where the PGA is 0.099g and the duration is 10.31sec. Regarding the fully filled state, the minimum displacement demand is attained for the earthquake M2, similar to the tank empty condition, but the maximum displacement demand is attained for the earthquake M7, where the PGA is 0.575g and the duration is 21.73sec. Additionally, it is observed that the displacement demand of the empty tank, in comparison to the fully filled tank, is generally smaller, except in three instances. These instances include the earthquakes M1-Mammoth Lakes-04, M4- Coyote Lake, and S4-Mammoth Lakes-01.

TABLE VI. MAXIMUM ROOF DISPLACEMENT

EQ ID	Displacement Demand (mm)	
	Empty	***Fully Filled***
M1	17.88	17.44
M2	1.951	4.243
M3	11.89	21.32
M4	13.08	10.98
M5	6.549	6.674
M6	17.52	15.46
S1	32.8	42.71
S2	44.66	50.59
S3	27.12	30.65
S4	68.21	50.97
M7	56.88	80.15

Fig. 8 and Fig. 9 illustrate the variation of displacement demand with PGA and Earthquake duration. Note that the PGA and duration axes are not scaled proportionally.

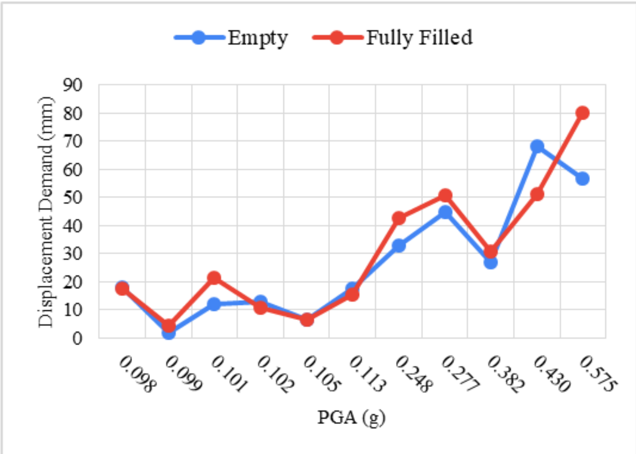

Fig. 8. Displacement demand variation with PGA

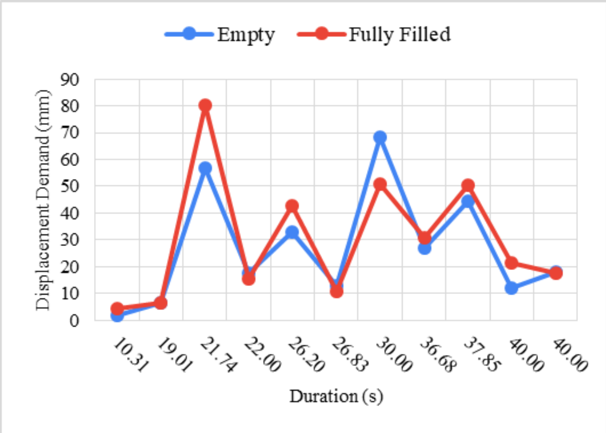

Fig. 9. Displacement demand variation with earthquake duration

It is observed that a direct relationship cannot be obtained for the displacement demand by considering PGA, magnitude, or duration individually.

B. *Base Shear*

The base shear of a structure subjected to an earthquake is the total lateral (horizontal) force or shear that is transmitted to the foundation of the building due to the seismic ground motion. The base shear is then distributed throughout the structure through lateral load-resisting elements like shear walls, moment frames, or bracing systems to ensure that the building can withstand the lateral forces and minimize damage. Accurate determination of the base shear is crucial in seismic design to ensure the safety of occupants and the structural integrity of the building during an earthquake.

Tab. 7 shows the maximum base shear values of all the cases considered. When evaluating the empty state of the tank, the maximum base shear was attained for the earthquake S4, while the minimum base shear was attained for the earthquake M2. Conversely, in the fully filled state, the minimum base shear was attained for the earthquake M2, similar to the tank empty condition, but the maximum base shear was attained for the earthquake M7. It was observed that the base shear increases in response to an increase in displacement demand unless the duration and PGA of the earthquake undergo significant variations.

TABLE VII. ABSOLUTE BASE SHEAR

EQ ID	**Absolute Base Shear (kN)**	
	Empty	***Fully Filled***
M1	99.41	89.01
M2	11.49	23.02
M3	65.66	115.29
M4	63.82	58.89
M5	29.18	29.93
M6	100.22	90.11
S1	155.00	180.05
S2	188.95	208.95
S3	166.21	155.94
S4	237.11	206.57
M7	209.38	257.91

C. *Plastic Hinge Formation*

Plastic hinges in structures are regions where significant plastic deformation or yielding occurs when the structure is subjected to loads, particularly during extreme events like earthquakes or strong winds. Plastic hinges are assigned to model these inelastic deformations accurately and it allows for the realistic representation of yielding and energy dissipation.

Based on the selected earthquakes, the formation of plastic hinges commenced when the PGA exceeded 0.105g. Tab. 8 includes the instance of forming plastic hinges. As shown in the table, the structure exhibited nonlinear behavior for earthquakes S1 to S4 and M7. Conversely, when the PGA was approximately 0.1g, the structure displayed linear behavior, resulting in the absence of plastic hinge formation. Fig. 10 shows the locations of plastic hinges.

TABLE VIII. PLASTIC HINGE FOMARTION

EQ ID	**Hinge Formation Start Time (s)**	
	Empty	***Fully Filled***
M1 to M6	Hinges aren't formed	Hinges aren't formed
S1	2.150	1.710
S2	5.815	5.900
S3	2.590	2.600
S4	5.655	8.500
M7	4.170	4.220

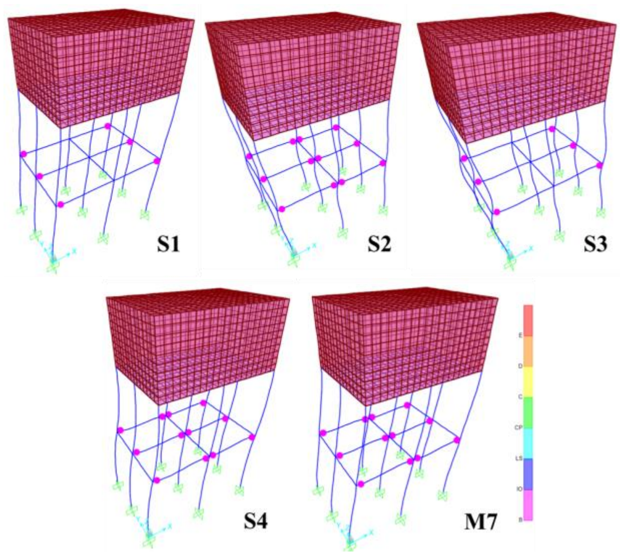

Fig. 10. Plastic hinge formation

D. *Natural Frequencies and Resonance*

The natural frequency of a structure is a fundamental property that characterizes its dynamic behavior. It represents the frequency at which a structure will naturally vibrate or oscillate when subjected to an external force or disturbance and then released. The natural frequency is primarily determined by the structure's mass and stiffness.

Modal analysis can be used to determine the natural frequencies and mode shapes of a structure. Tab. 9 highlights the natural frequencies corresponding to the first ten modes. It is observed that the natural frequency of the fully filled tank is always lower than the corresponding natural frequency of the empty tank. Determination of natural frequencies is essential for ensuring that the structure doesn't experience resonance, which can lead to structural damage or instability.

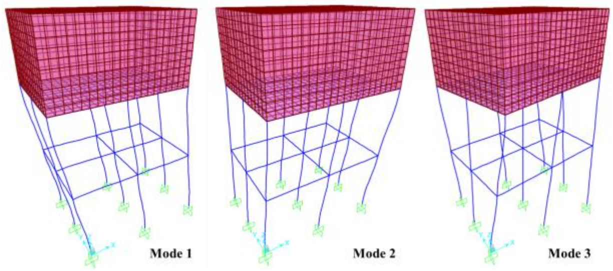

Fig. 11. First three mode shapes of the empty tank

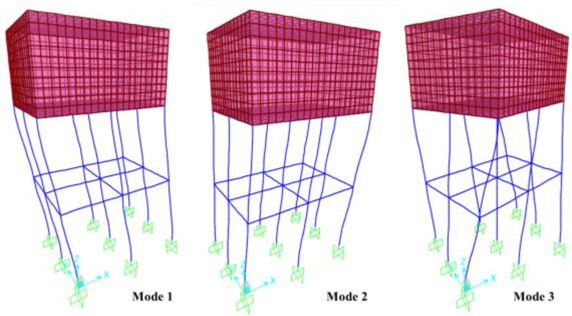

Fig. 12. First three mode shapes of the fully filled tank

TABLE IX. NATURAL FREQUENCIES OF THE TANK

Mode No.	Natural Frequency (Hz)	
	Empty	*Fully Filled*
1	0.71	0.49
2	0.73	0.50
3	0.88	0.57
4	4.57	4.53
5	4.59	4.54
6	5.05	4.99
7	8.56	8.56
8	9.20	9.20
9	13.02	12.70
10	13.64	13.64

Fourier analysis is utilized to analyze the individual frequencies of an earthquake by decomposing the ground motion into its constituent frequency components. In order to assess the resonance effect, the dominant Fourier frequencies of individual earthquakes can be compared with the natural frequencies of the structure. The dominant frequencies of the selected earthquakes obtained using the Seismosignal software [14] are recorded in Tab. 10. The first three mode shapes of empty and fully filled tanks are shown in Fig. 11 and 12, respectively. Frequency spectrum of Coyote Lake earthquake is shown in Fig. 13 as a representative case.

It was observed that the dominant frequencies of M1, M4, and S4 earthquakes correspond with the natural frequencies of the first several modes of the empty tank. Specifically, the frequency of 0.98 Hz in M1 nearly matches with the natural frequency of 0.88 Hz, while the frequency of 5.05 Hz in M4 corresponds with a natural frequency of 5.05 Hz. Additionally, the frequency of 4.57 Hz in S4 corresponds with a natural frequency of 4.57 Hz. Therefore, the maximum displacement demands obtained from the analyses contain the effects of resonance.

TABLE X. FREQUENCY CONTENT OF THE EARTHQUAKES

EQ ID	Dominant Frequencies (Hz)				
	1	*2*	*3*	*4*	*5*
M1	0.98	1.37	2.10	2.98	3.83
M2	2.83	6.54	11.91	12.79	13.62
M3	1.68	4.08	4.25	4.96	6.01
M4	1.20	1.61	2.98	4.42	5.05
M5	1.61	2.10	2.78	3.71	6.88
M6	1.15	1.42	1.64	1.78	2.12
S1	1.03	1.42	1.56	2.10	3.71
S2	1.00	1.25	1.34	1.86	2.10
S3	4.13	4.44	5.52	5.76	6.40
S4	1.34	1.83	2.42	3.34	4.57
M7	1.17	1.44	1.64	2.37	4.13

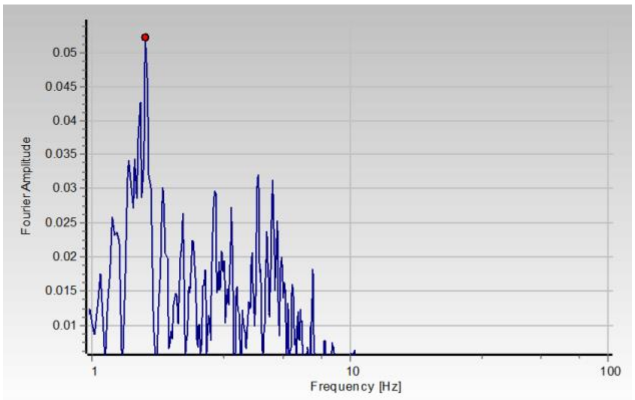

Fig. 13. Frequency content of the Coyote Lake earthquake

IV. CONCLUSION

The following conclusions can be drawn based on the results of the nonlinear time history analysis of the elevated water tank.

- The increased displacement demand of the empty tank, compared to the fully filled tank in the three specific cases of earthquake events (M1- Mammoth Lakes-04, M4- Coyote Lake, and S4-Mammoth Lakes-01), highlights the presence of a resonance effect.
- The displacement demand is not solely determined by a single factor, but rather it is influenced by a combination of factors, including peak ground

acceleration (PGA), duration, and earthquake magnitude.

- As expected, magnitude of the base shear tends to rise as the displacement demand of the structure increases.
- The observed lower natural frequency in the fully filled tank compared to the empty tank indicates that the added mass from the liquid content has a more significant effect on the tank's dynamic behavior compared to any potential increase in stiffness due to the liquid.
- While plastic hinges are formed during earthquakes with a PGA of up to 0.5g, it is essential to note that the structural integrity of the tank remains intact.
- The absence of plastic hinge formation under seismic conditions with a PGA around 0.1g indicates that, despite this type of tanks in Sri Lanka not being explicitly designed to withstand seismic forces, they exhibit a robust resistance to earthquakes of this magnitude. Consequently, it is evident that such tanks are unlikely to fail during the aforementioned seismic events.

In future research, the seismic analysis can be extended to evaluate the designed water tank's capacity under different earthquake records having higher PGA, various frequency contents, durations, and magnitudes. The effects of sloshing in the case of partially filled tanks can also be incorporated in the analysis. This would provide valuable insights into elevated water tanks' seismic resilience under more severe seismic conditions.

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